Traffic Jams in a Lattice-Gas Model

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We propose a simple lattice-gas model characterized by two states of atoms, the "thermalized" state, which is the same as in the standard lattice-gas model, and the "running" state, where the atoms jump in one direction only. The model exhibits the existence of "traffic jams" (bunching of thermalized atoms in compact groups), the nonlinear dependence of mobility on the jump probability, and the hysteresis.

1. INTRODUCTION

Driven diffusive systems belong to the simplest models of nonequilibrium statistical mechanics. These systems are characterized by a locally conserved density, and a uniform external field sets up a steady mass current. The systems of this class have a wide application area in modeling of charge and mass transport in solids. Last years the driven diffusive models are used in tribology, where the driving force emerges owing to motion of one of two substrates separated by a thin atomic layer.

In the context of tribology, the generalized Frenkel-Kontorova (FK) model has been studied recently[1]. In this model, a one- or two-dimensional atomic system is placed into the external periodic potential, and the atomic current j in response to the dc driving force F is studied by solution of Langevin motion equations. The simulation showed that the function $j(F)$ exhibits hysteresis: when the force increases, the system goes from the low-mobility regime to the high-mobility state, where all atoms move with

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almost maximum velocity. But if the force is then decreased, the high-mobility state persists till quite small values of F, and then jumps abruptly to the low-mobility state. Besides, during the transition the atoms have a tendency to be organized in compact groups of two different types, one consisting only of slowly moving atoms (which resemble "traffic jams"), and another of "running" atoms moving with the maximum velocity.

However, the FK model studied in[1] is too complicated to be studied in all details. For this reason it is convenient to develop a more simple model which will capture the most important features of the FK model. Although in this case we lose the possibility of exactly predicting the characteristics of a real physical object, a new sight on the problem could help to understand the behavior of more realistic and complicated models.

Microscopically, the driven diffusive system may be modeled as a lattice gas (LG) where particles occupy the sites with at most one particle per site. The atoms jump stochastically to vacant nearest-neighbor sites, and the external field biases jumps in the positive x direction[2]. This model is known as the partially asymmetric exclusion model (ASEP). Driven lattice gases with hard-core repulsion traditionally are used to describe hopping conductivity in solids. The extreme case of this model, the totally asymmetric exclusion model has been solved exactly[3].

In the model mentioned above, an atom jumps to the right with the probability α and to the left with the probability $1 - \alpha$, where $1/2 < \alpha \leqslant 1$. The totally asymmetric exclusion model corresponds to the case of $\alpha = 1$. Recalling the FK model, the probability of a jump to the right at small dc forces is approximately equal to

$$\alpha \approx (1 + e^{-aF/T})^{-1} \tag{1}$$

where T is the temperature and a is the period of the external potential. At a high driving force, $F \geqslant F_f$, where $F_f \approx \pi \varepsilon / a$ for the sinusoidal substrate potential (here ε is the height of the potential), the atom can jump only to the right, because the potential barriers are totally degraded at these forces. In the lattice gas model, this situation corresponds to the $\alpha = 1$ case. Thus, the parameter α plays the role of the driving force of the FK model.

The FK model studied in ref. 1 has, however, one more aspect connected with the existence of external damping in Langevin equations. When the damping coefficient η is large, the atom after the jump stops in the new potential well. But if η is small, there exists a threshold force $F_b (F_b \approx \eta \sqrt{m\varepsilon}$, m being the atomic mass) such that at $F > F_b$ the atom after the jump does not stop but continues to move till it meets a stopper, e.g., a thermalized atom in front of itself. To incorporate this feature into the lattice gas model, we assume that an atom may be in two different

states, in the "thermalized" state, in which it jumps as usual in the LG model, and in the "running" state, in which the atom always jumps to the right provided the right-hand site is empty. Thus, the new model incorporates the features of both partially and totally asymmetric models.

The important aspect of the model introduced in the present work, is that an atom can change its state from the thermalized state to the running state and vice versa: the thermalized atom becomes in the running state after the jump to the right, and the running atom becomes thermalized after a collision with another thermalized atom. Models with multiple states belong to cellular automata type models which are used last time in simulation of highway traffic (e.g., see recent survey[4]). Because the atoms in the model under consideration behave like vehicles in a one-lane road, the present model may also be considered as a new variant of the traffic-jam model.

We consider three slightly different variants of the evolution rules, and call the corresponding models as the model **A**, **B**, and **C** respectively. The simplest model **A** already exhibits the nonlinear mobility, irreversibility, and shows the appearance of "traffic jams" (bunching of thermalized atoms). A more realistic model **C**, additionally, has a hysteresis which resembles that of the FK model. All three models are studied with Monte Carlo technique as well as analytically within a mean-field approximation.

2. MODEL A

The simplest model studied in the present work is defined as follows. Consider a one-dimensional lattice of length M with the periodic boundary conditions. Each site is either occupied by one atom or is empty. Let N is the total number of atoms, and the dimensionless concentration is defined as $\theta = N/M$. Each atom may be in one of two states, in the thermalized state or in the running state. The system evolves in time according to the sequential dynamics, i.e., particles jump independently and randomly according to the following rules:

AI. At each time step $t \to t + 1$, one chooses at random a site i.

AII. If this site is occupied by a thermalized atom, it jumps to the site $i + 1$ (if this site is empty) with the probability α, or it jumps to the site $i - 1$ (if the left-hand site is empty) with the probability $1 - \alpha$ as in the partially asymmetric exclusion model. After the jump to the left the atom remains in the thermalized state, while *after the jump to the right the atom becomes in the running state.*

AIII. If the atom in the chosen site i is in the running state, it jumps to the right provided the right-hand site is empty, and remains in the

running state. Otherwise, if the site $i+1$ is not empty, the atom in the site i remains in the running state if the right-hand site is occupied by the running atom, or *becomes thermalized if the site $i+1$ is occupied by the thermalized atom*.

Thus, the model A is characterized by two parameters, θ and α. A typical picture of system evolution started from a random distribution of thermalized atoms, is shown in Fig. 1. As seen, from the very beginning the system splits into compact domains of thermalized and running atoms. The immobile (thermalized) domains are characterized by the local atomic concentration $\theta_s = 1$. Below we will call these immobile domains by jams. The
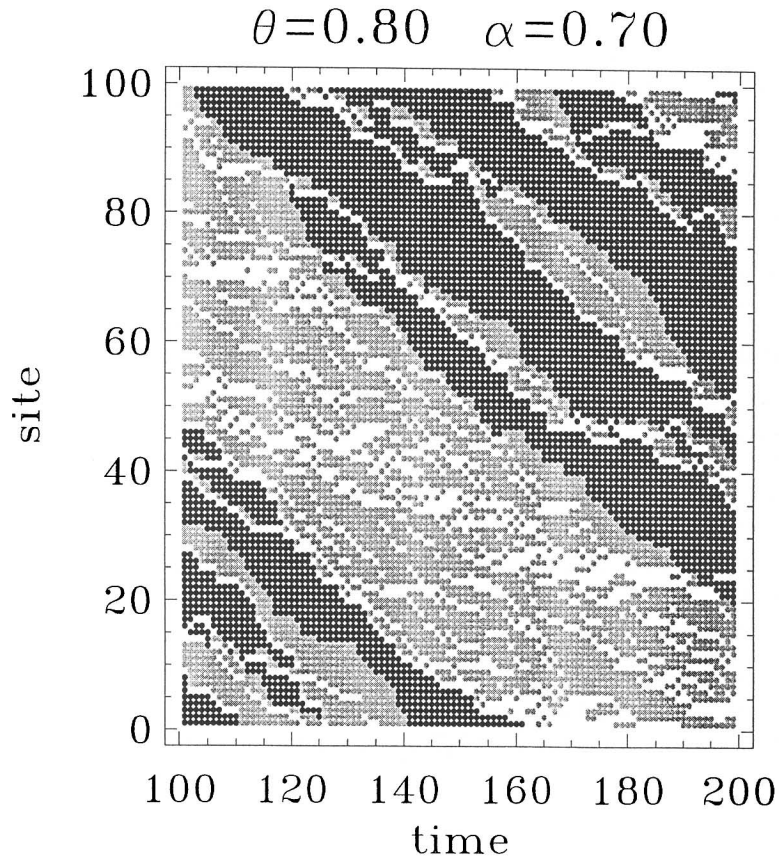

Fig. 1. Evolution of the model A. Thermalized atoms are shown as black circles, and running atoms, as grey circles. Time is measured in Monte Carlo attempts per site. The system size is $M = 10^3$, $\theta = 0.8$, and $\alpha = 0.7$.

jams are separated by running domains (RD's) characterized by a local concentration $\theta_r < \theta$.

To characterize the system state, let us introduce the "mobility" B as the ratio of the number of running atoms N_r to the total number of atoms N, $B = N_r/N$. The dependences of B on α for different values of θ are shown in Fig. 2.

The behavior of the running domains of the present model should remind that of the totally asymmetric exclusion model with open boundary conditions.[3] Recall that in the totally asymmetric exclusion model all atoms are of the running type, and an atom may come into the chain from the left-hand side (if the most left site is empty) with a probability α, and the atom from the most right site may leave the chain with a probability β. Note, however, that the running domains of the present model are not equivalent to the totally asymmetric exclusion model because of different (more simple in the present work) boundary conditions. According to the
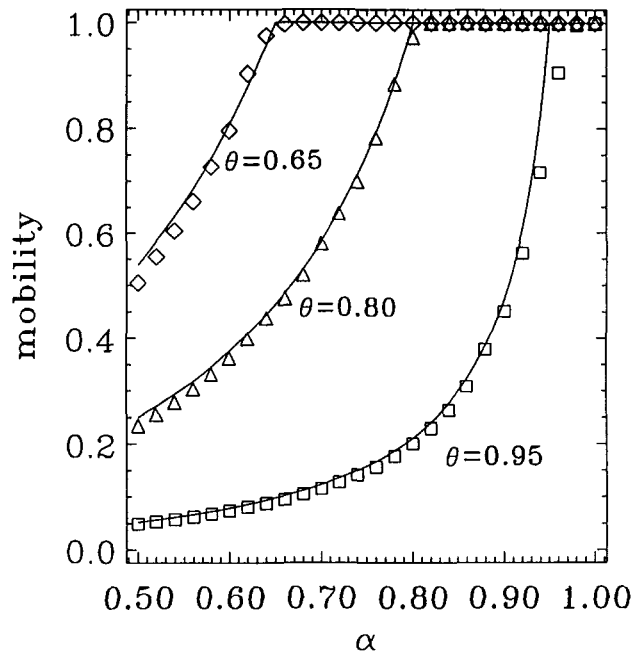

Fig. 2.   Mobility B as function of the jump probability α for different values of θ: $\theta = 0.65$ (diamonds), $\theta = 0.80$ (triangles), and $\theta = 0.95$ (squares). Each data point is an average over 5×10^3 attempted jumps per site, the averaging was started after 5×10^3 MC steps. The data were averaged additionally over 28 independent runs. The system size is $M = 10^3$. Solid curves are the predictions of Eq. (6).

rules accepted for the model A, the most left site of any RD is always empty (this is clearly seen in Fig. 1). Therefore, the running domain grows from its left-hand side with the rate α owing to injection of new atoms from the left-hand-side neighboring jam. At the right-hand side of the RD, the atom which occupies the most right site of the RD, leaves the RD and joins itself to the neighboring right-hand-side jam. Thus, the RD shorters from the right-hand side with the rate p_r, where p_r is the probability that the most right site of the RD is occupied. Clearly, in the steady state $p_r = \alpha$.

To calculate B, let us assume that there is only one jam in the chain. Let this jam has the length s. Because the local concentration in the jam is $\theta_s = 1$, we can apply the following simple arithmetic,[1] is

$$s + N_r = N, \qquad s + M_r = M \tag{2}$$

where M_r is the length of the running domain. Taking into account that $N_r = M_r \theta_r$ and $N = M\theta$, we obtain

$$s = M \frac{\theta - \theta_r}{1 - \theta_r} \tag{3}$$

so that the mobility is equal to

$$B = \frac{\theta_r (1 - \theta)}{(1 - \theta_r)\, \theta} \tag{4}$$

Evidently, Eq. (4) is valid as well for the steady state with any number of jams provided θ_r corresponds to the mean atomic concentration in RD's.

Neglecting by a possible deviation of the RD concentration at its right-hand side from the mean value θ_r, we may take approximately

$$p_r \approx \theta_r \tag{5}$$

and taking then into account that $p_r = \alpha$ in the steady state, we finally come to the expression

$$B \approx \frac{\alpha(1 - \theta)}{(1 - \alpha)\, \theta}, \qquad \alpha < \theta \tag{6}$$

For $\alpha > \theta$ the jams disappear at all, and $B = 1$ in the steady state. The dependences (6) which are shown by solid curves in Fig. 2, describe the simulation results with a good accuracy.

The model A described above is similar to the Nagel-Schreckenberg (NS) "minimal" model of real traffic.[5] The present model differs from the

NS one in two aspects: first, we are using the sequential dynamics contrary to the "parallel update" of the NS model, and second, the low-velocity state of our model corresponds to thermalized atoms while in the NS case it corresponds to immoxbile cars. Both these features are natural for the system of atoms in contact with thermal bath.

If we introduce the atomic flux as $j = \theta v$, where v is the average velocity of atoms (this is the standard expression for the LG model), then the main issue of the traffic theory, the fundamental diagram (flux versus density) takes the trivial form $j = \theta(1 - \theta)$. However, in a spirit of the FK model, where all atoms in the running domain move simultaneously, it is more natural to define the "flux" as $j = \theta B$, where the "mobility" B was introduced above. For this definition of j, the fundamental diagram takes the triangular shape, $j = \theta$ for $\theta < \alpha$ and $j = (1 - \theta)\alpha/(1 - \alpha)$ for $\theta > \alpha$, which is similar to that of real traffic[4].

Now let us dwell on the steady state of the present model in more details. A jam of length s loses atoms from its right-hand side with the
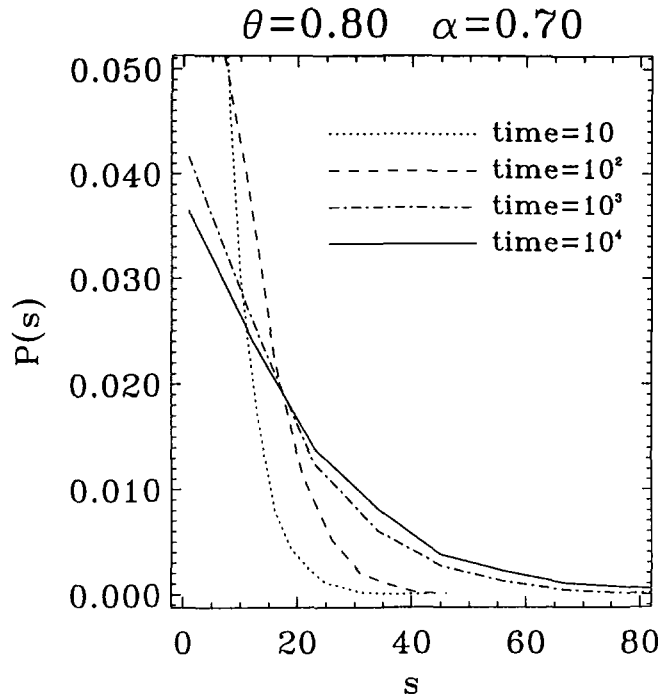

Fig. 3. Distribution of jam sizes at different times: $t = 10$ (dotted curve), $t = 10^2$ (dashed curve), $t = 10^3$ (dot-dashed curve), and $t = 10^4$ (solid curve). Chain's length is $M = 10^3$, $\theta = 0.8$, and $\alpha = 0.7$. The histograms were averaged over 100 independent runs.

rate α, and it receives new atoms to the left-hand side with the rate p_r. These two rates are equal one another in the steady state, so in average $\langle \dot{s}(t) \rangle = 0$. But because of randomness of joining and losing events, the value $s(t)$ will exhibit random walks, i.e., at long times, $t \gg 1$, $t' \gg 1$, and $|t - t'| \gg 1$, $s(t)$ should behave according to the equation

$$\langle [s(t) - s(t')]^2 \rangle \approx 2\alpha |t - t'| \tag{7}$$

Thus, at $\alpha < \theta$ *the infinite system has no steady state at all.* Indeed, when a jam reaches the size $s = 0$, it disappears forever, while the motion of $s(t)$ to higher values is not bounded in the infinite system. The distribution of jam sizes $P(s)$ continuously changes with time shifting to larger and larger values as shown in Fig. 3, so that instead of the name "steady state" it is more reasonable to use the name "coarsening state". But the mobility of the coarsening state does not change with time as seen from Fig. 4. According to Eq. (6), B is determined by the system parameters only and does not depend on the distribution $P(s)$.
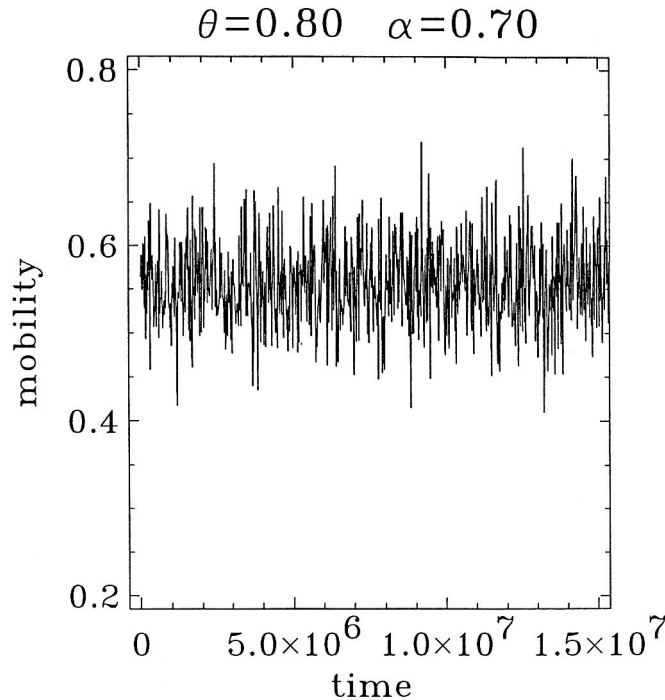

Fig. 4. Typical dependence $B(t)$ for a long-time simulation $(t > 10^7)$. The system size is $M = 10^3$, $\theta = 0.8$, and $\alpha = 0.7$.

On the other hand, a *finite* system does have a true steady state, and this state corresponds to the absence of jams at all for any model parameters. Indeed, in a finite system the maximal jam's size is restricted by the value given by Eq. (3), and after a long enough time τ all jams should finally disappear. A typical dependence $B(t)$ is shown in Fig. 5. The average value of τ can be estimated from the relation $s^2 = 2\alpha\langle\tau\rangle$, that leads to the expression

$$\langle\tau\rangle = M^2 \frac{(\theta - \alpha)^2}{2\alpha(1 - \alpha)^2} \tag{8}$$

The distribution $P(\tau)$ of the "first passage times" to reach the running state is shown in Fig. 6 together with the estimation (8).

The transition to the running state in the model A is of second order in the sense that the current changes continuously at the point $\alpha = \theta$. However, the simplest model exhibits a "trivial" hysteresis: as far as *all*

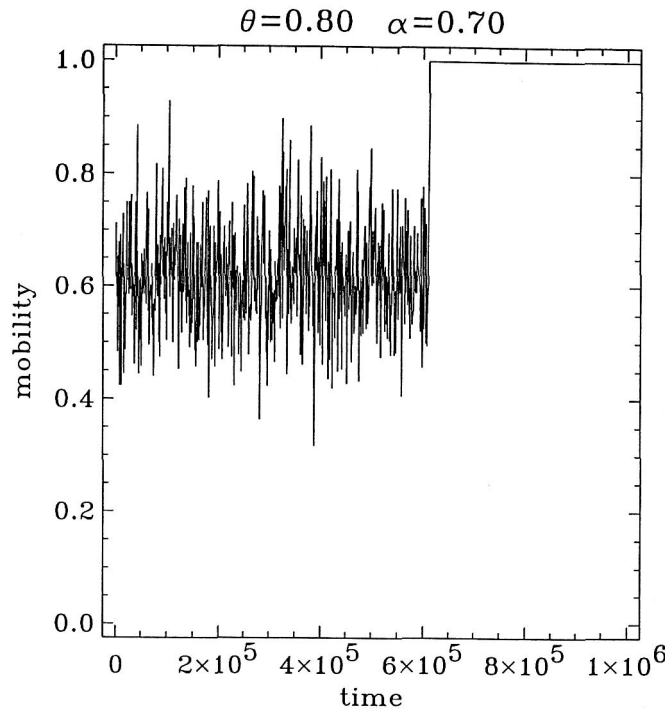$\theta = 0.80 \quad \alpha = 0.70$

Fig. 5. Dependence $B(t)$ in a finite system: in the model A the transition to the running state is irreversible. Chain's length is $M = 300$, $\theta = 0.8$, and $\alpha = 0.7$.
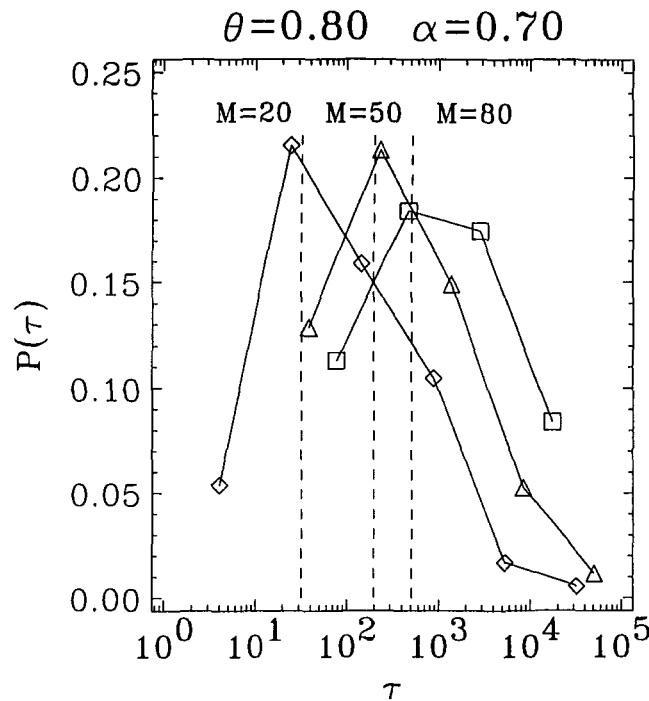
$$\theta = 0.80 \quad \alpha = 0.70$$

Fig. 6. Distribution of times of reaching of the running state in chains of different lengths: $M = 20$ (diamonds), $M = 50$ (triangles), and $M = 80$ (squares). The system parameters are $\theta = 0.8$ and $\alpha = 0.7$. The histograms were averaged over 200 independent runs. Broken lines indicate the times predicted by Eq. (8).

atoms came into the running state, this state then remains unchanged forever, even if α is changed back to lower values. In this sense the transition may be considered as of first order.

3. MODEL B

Keeping in mind to describe qualitatively the behavior of the FK model mentioned at the beginning of the article, let us improve the model A to make it more realistic, allowing the running atoms to change spontaneously their state back to the thermalized state with some probability γ. Indeed, in the model A the transition of an isolated atom to the running state was irreversible. This is true for large driving forces $F > F_f$, i.e., for the $\alpha = 1$ case. But at smaller forces, $F < F_f$, or $\alpha < 1$, an isolated atom must have a nonzero probability to come back to the thermalized state. For this

reason we now slightly modify the third rule of system evolution in the following way:

BIII. If the atom in the chosen site i is in the running state, it comes to the thermalized state with the probability $\gamma(\gamma < 1)$, or evolves according to the rule AIII with the probability $1 - \gamma$.

The dependences $B(\alpha)$ for different values of γ are shown in Fig. 7. The model **B** has the true steady state, and $B < 1$ for any model parameters provided $\gamma > 0$.

To describe the model **B** analytically, we use the mean-field (MF) approach and suppose that all running domains are characterized by the same concentration θ_r. Besides, suppose also that the probability of occupation of the most right site of the running domain is equal θ_r as above in Eq. (5). With these assumptions, a jam of length s increases $(s \to s + 1)$ with the rate θ_r, and decreases $(s \to s - 1)$ with the rate α.
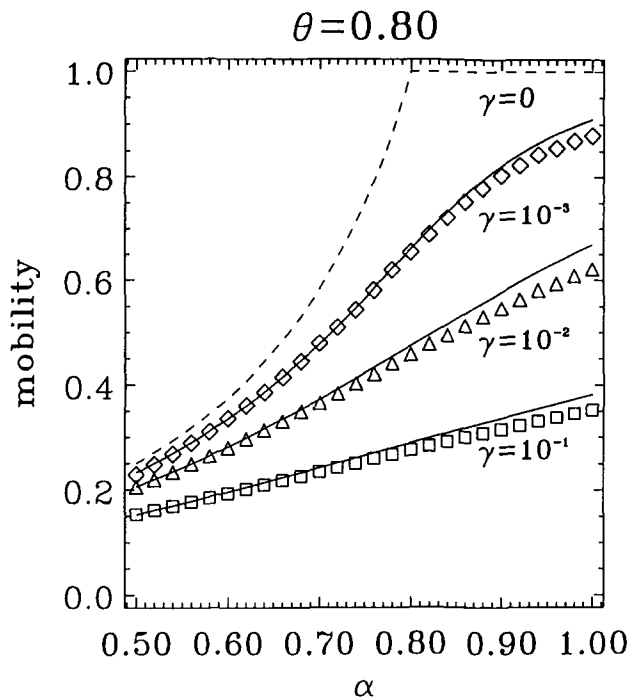

Fig. 7. B versus α for the model **B** for different values of γ: $\gamma = 10^{-1}$ (squares), $\gamma = 10^{-2}$ (triangles), and $\gamma = 10^{-3}$ (diamonds). $\theta = 0.8$, and the system size and statistics are as in Fig. 2. The dependences (13) are shown by solid curves. The $\gamma = 0$ dependence (6) is also shown by broken curve.

Denoting by $n_s(t)$ the number of jams of length s at time t, we can write the following kinetic equation for $n_s(t)$,

$$\dot{n}_s = -\alpha n_s - \theta_r n_s + \theta_r n_{s-1} + \alpha n_{s+1}, \qquad s \geqslant 2 \tag{9}$$

In the continuum limit, $n_s \gg 1$, Eq. (9) reduces to the diffusion equation

$$\dot{n}(s, t) = (\alpha \nabla_s^2 + \mu \nabla_s) n(s, t) \tag{10}$$

with the "drift" coefficient $\mu \equiv \alpha - \theta_r$. In the model **B** we always have $\mu > 0$ because n_1 is finite due to creation of new thermalized atoms in contrast with the model **A**, where it was $n_1 \to 0$ in the limit $t \to \infty$.

Equation (9) has a simple steady state solution $n_s = n_1(\theta_r/\alpha)^{s-1}$. The distribution of jam's sizes is described by the expression

$$P(s) \equiv \frac{n_s}{J} = \frac{\alpha - \theta_r}{\alpha} \left(\frac{\theta_r}{\alpha}\right)^{s-1} \tag{11}$$

where $J = \sum n_s = n_1 \alpha / (\alpha - \theta_r)$ is the total number of jams. As seen from Fig. 8, Eq. (11) describes the simulation data quite well.

The total number of thermalized atoms is equal to

$$N_s = \sum s P(s) = n_1 \left(\frac{\alpha}{\alpha - \theta_r}\right)^2 \tag{12}$$

Applying now the same arithmetic as above in Eq. (2), we can find the mobility,

$$B = (1 + p)^{-1} \tag{13}$$

and the average concentration in the running domains,

$$\theta_r \equiv \frac{N_r}{M_r} = \frac{\theta}{1 + p - p\theta} \tag{14}$$

which depend on the parameter p defined as

$$p = \left(\frac{n_1}{N_r}\right)\left(\frac{\alpha}{\alpha - \theta_r}\right)^2 \tag{15}$$

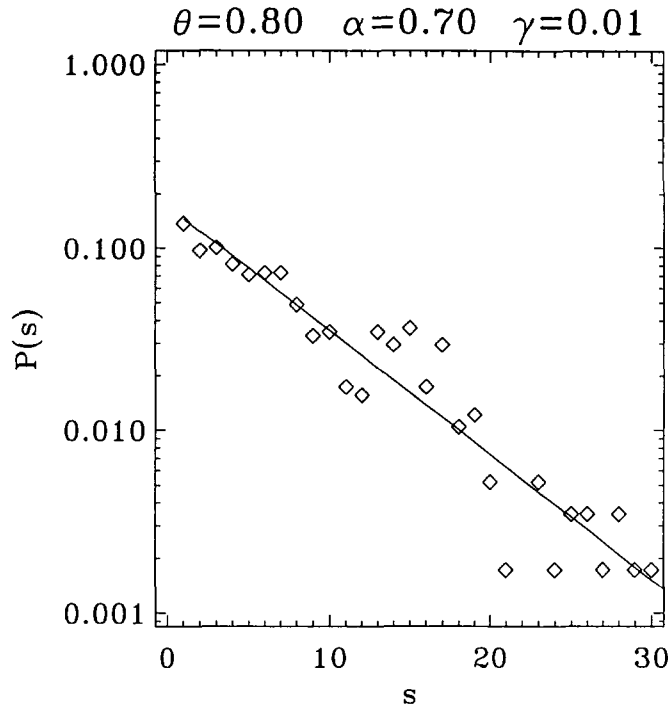$$\theta = 0.80 \quad \alpha = 0.70 \quad \gamma = 0.01$$

Fig. 8. Distribution of jam's sizes in the model **B** for $\gamma = 0.01$. Chain's length is $M = 10^4$, $\theta = 0.8$, $\alpha = 0.7$, and the simulation time is $t = 10^4$. The histogram is an average over 100 independent runs. Line corresponds to the estimation of Eq. (11).

To complete the set of equations, we have to write a kinetic equation for $n_1(t)$ which plays a role of boundary condition for Eqs. (9). It may be written as follows:

$$\dot{n}_1 = \gamma N_r + \alpha n_2 - \alpha(1 - \theta_r) n_1 - \theta_r n_1 \tag{16}$$

The first term in the right-hand side of Eq. (16) describes the creation of thermalized atoms from the running atoms, and the other terms have the same meaning as in Eq. (9) except that now $n_0 = 0$ and the rate of disappearing of one-atomic jams is not α but $\alpha(1 - \theta_r)$, because these jams are emerging mostly inside the running domains, and the condition that the next-neighboring site to the right of the one-atomic jam is empty, is not valid anymore.

The steady state solution of Eq. (16) is

$$n_1 = \gamma N_r / \alpha(1 - \theta_r) \tag{17}$$

Substituting (17) into Eqs. (14–15), we obtain an equation on θ_r,

$$(\theta - \theta_r)(\alpha - \theta_r)^2 (1 - \theta_r) - \gamma\alpha\theta_r(1 - \theta) = 0 \qquad (18)$$

which may easily be solved numerically. The MF dependences $B(\alpha)$ are presented in Fig. 7.

The MF approach used above, was based on the assumption that all running domains are characterized by the same local concentration θ_r. This assumption is true for the model **A**, where in the limit $t \to \infty$ the average size of RD is growing unboundedly. But for the model **B**, where n_1 must have a finite value, this assumption is not valid, this is clearly seen in Fig. 9. Most of RD's are characterized by simple rational concentrations like $\frac{1}{2}$, $\frac{2}{3}$, etc. In spite of this, the agreement of the MF approach with simulation data is surprisingly good.

Last, note that the model **B** has no irreversibility anymore, the $\gamma \neq 0$ condition totally kills the hysteresis presented in the model **A**.
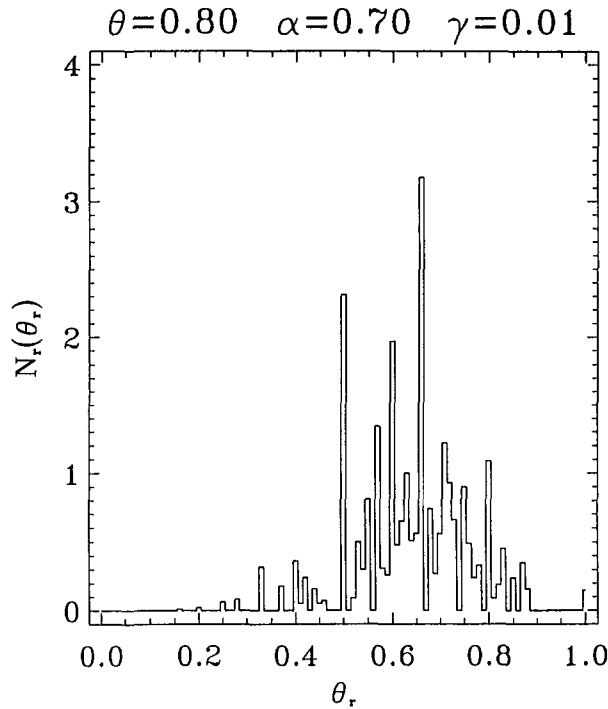

Fig. 9. Number of running atoms in running domains with a given concentration θ_r as function of θ_r. The parameters are as in Fig. 8.

4. MODEL C

In the model **B** the spontaneous transition of an atom from the running state to the thermalized state does not depend on the state of surrounding atoms. This is true for the transition of an isolated atom, but is too crude approximation for the FK system which we are trying to model. Indeed, when an atom inside a RD became thermalized, it will be immediately pulled back to the running state by the running atoms behind it. This "inertia" effect can not be described rigorously in the framework of a lattice-gas model. But let us try to simulate this effect qualitatively, modifying the third evolution rule in the following way:

CIII. In the case when a randomly chosen site i is occupied by a running atom and the site $i + 1$ is occupied by a thermalized atom, we will count the total number s of thermalized atoms in the compact jam to the right of the bond $i - (i + 1)$, and the number r of running atoms behind this bond in the compact running block, i.e., s is the distance from the bond $i - (i + 1)$ to the first empty site in the positive x direction, and r is the same distance in the negative x direction. Then the system is updated according to the following rule: if $r < s$, all $r + s$ atoms become thermalized, otherwise (if $r \geqslant s$), all $r + s$ atoms "belonging" to the $i - (i + 1)$ bond, become running. In all other cases the system evolves similarly to the model **B**, i.e., if the site $i - 1$ is empty, the atom either comes to the thermalized state with the probability γ, or remains in the running state with the probability $1 - \gamma$, jumping to the right provided the site $i + 1$ is not occupied.

Besides, let us make one more improvement of the model. As was mentioned above, the rate of spontaneous transition to the thermalized state should depend on the external force F, i.e., on the inclination of the periodic substrate potential; it must be zero for $F > F_f$ and it has to increase with F decreasing in the region $F < F_f$. In the lattice-gas model, this means that γ should be zero at $\alpha = 1$ but has to increase when α decreases. For concreteness, let us take the simple expression

$$\gamma = \gamma_0 (1 - \alpha)^2 \tag{19}$$

where γ_0 is a model parameter. We took the square dependence in order to avoid an unphysical peculiarity in the $\alpha \to 1$ limit.

The simulation results for the model **C** are presented in Fig. 10. One can see that the $B(\alpha)$ dependences of this model are very close to those of the model **A** (or the model **B** with a very small γ). The explanation of this behavior is simple: in the model **C** one-atomic jams have practically no
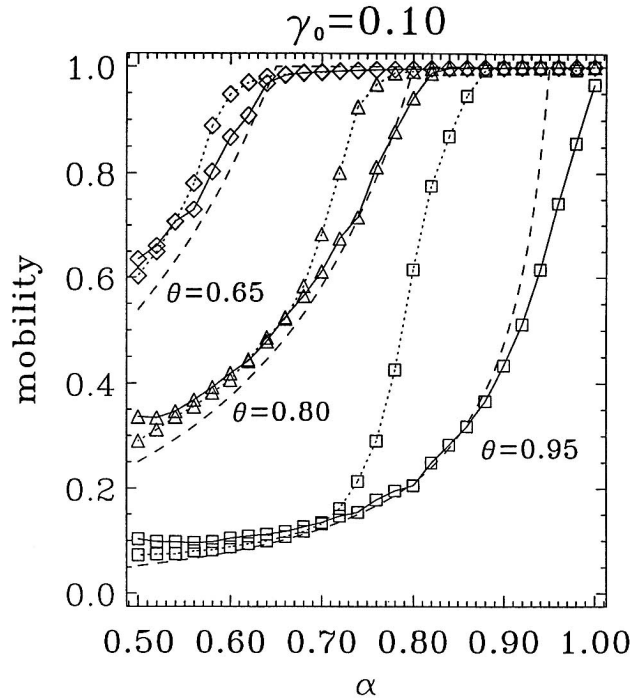
Fig. 10. Mobility $B(\alpha)$ for the model C for different values of θ: $\theta = 0.65$ (diamonds), $\theta = 0.80$ (triangles), and $\theta = 0.95$ (squares). For each θ the simulation was started from the random configuration of thermalized atoms at $\alpha = 0.5$, then the final configuration of the previous step was used as the initial configuration for the next value of α. Solid curves correspond to the increasing of α, and the dotted curves, to its decreasing. Each data point is an average over 1.5×10^3 attempted jumps per site, the averaging was started after 1.5×10^3 MC steps. The data were averaged additionally over 30 independent runs. The system size is $M = 10^3$, and $\gamma_0 = 0.1$. The dashed curves correspond to Eq. (6) of the model A.

chances to survive, and this results in a very small values of n_1 just as it was in the model A.

The main new feature of the model C is that it has a nontrivial hysteresis similarly to that of the FK model.[1] When α decreases starting from the $\alpha = 1$ value, the state without jams (the running state, or RS) survives at values of α lower than θ before the RS jumps back to the state with jams (the jam state, or JS). As seen from Fig. 10, the width of the hysteretic loop depends on the concentration θ, for higher values of θ the loop is wider. Besides, the amplitude of hysteresis depends on the rate of α variation as seen in Fig. 11. A slower α is changed, a more narrow is the hysteretic loop, so that for the adiabatically slow variation of α the
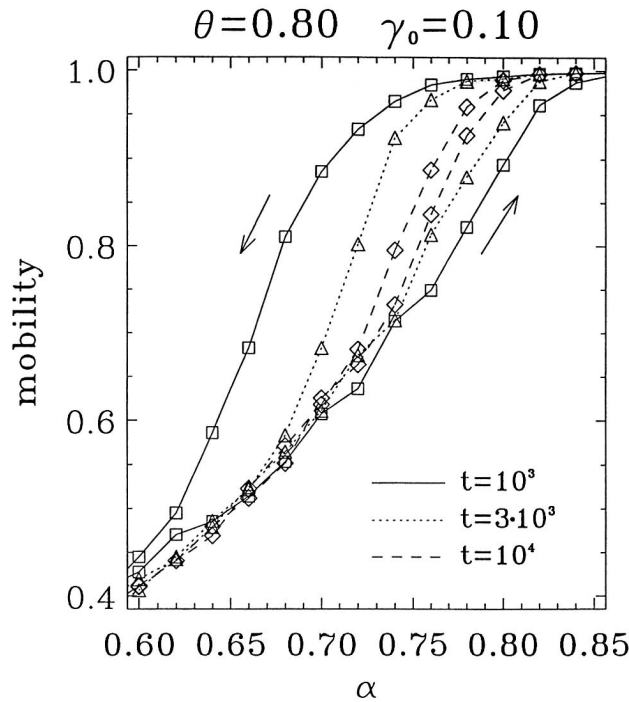
$$\theta = 0.80 \quad \gamma_0 = 0.10$$

Fig. 11. Hysteresis in the model C for different simulation times: squares and solid curve, for an averaging over 0.5×10^3 MC steps per site, the averaging started after 0.5×10^3 steps, triangles and dotted curve, for an averaging over 1.5×10^3 steps, the averaging started after 1.5×10^3 steps, and diamonds and dashed curve, for an averaging over 5×10^3 MC steps, the averaging started after 5×10^3 steps. Chain's length is $M = 10^3$, $\theta = 0.8$, $\gamma_0 = 0.1$, the data were averaged additionally over 30 independent runs.

hysteresis should disappear at all. This means that the running state of the system at $\alpha < \theta$ is in fact a metastable state characterized by a finite lifetime τ_r. The distribution of lifetimes τ_r of the running state is shown in Fig. 12.

To estimate a mean value of τ_r analytically, let us consider a state with a single jam of size s at time t. This jam will be killed in the next MC step $t + 1$, if just behind it there is a compact block of running atoms of a size $r \geqslant s$. In the RS the probability to have such a block of running atoms is θ^s. Thus, at the time $t + 1$ the s-atomic jam will disappear with the probability θ^s, and it will survive till the next time step with the probability $1 - \theta^s$. Therefore, in average the s-atomic jam will survive if $1 - \theta^s \geqslant \theta^s$, or $s \geqslant s_0$, where the critical size so is equal to

$$s_0 = -\frac{\ln 2}{\ln \theta} \tag{20}$$

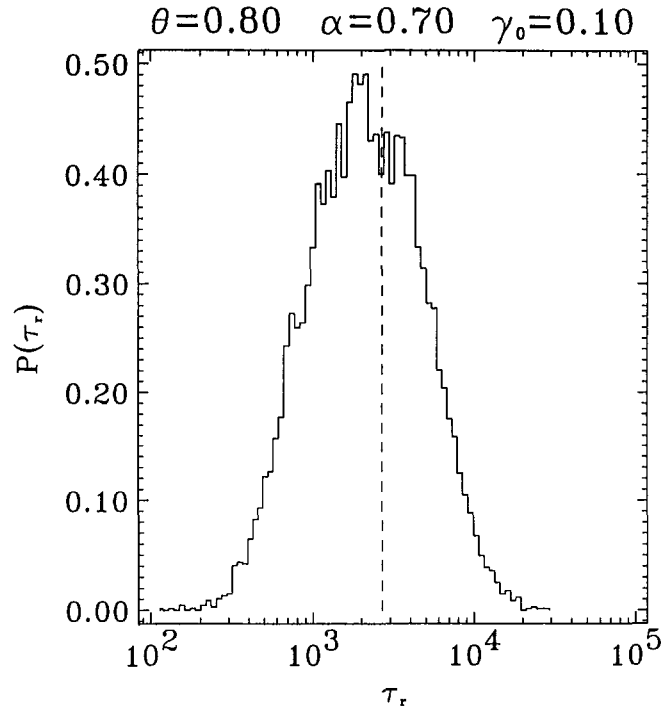$\theta = 0.80 \quad \alpha = 0.70 \quad \gamma_0 = 0.10$

Fig. 12. Distribution of times τ_r of transition from the totally running state to a state with jams for the model C. The system size is $M = 10^3$, $\theta = 0.8$, $\alpha = 0.7$, and $\gamma_0 = 0.1$. The hystogram was calculated with 8×10^3 points, each simulation was started from the initial configuration of running atoms, then it was "equilibrated" at $\alpha = 1$ during the time 10^3 MC steps, after that α was abruptly decreased to the value $\alpha = 0.7$, and the simulation was stopped when B reaches the value $B = 0.9$ for the first time. Broken line shows the prediction of Eq. (21).

Now let us suppose that at the time $t - 1$ the chain was in the running state, and calculate the probability that at the time t the chain has a jam of the size s_0. For one MC step, the number of newly created thermalized atoms is γN (recall $N_r = N$ in the RS), so that the probability for a given site to be occupied by one thermalized atom, is $\gamma N/M = \gamma\theta$. Therefore, the probability that at a given place it will appear the s_0-atomic jam, is $(\gamma\theta)^{s_0}$. Taking now into account that for the transition to the JS the chain may have only one jam, we obtain that in the chain of length M the probability of the RS → JS transition per one MC step is $M(\gamma\theta)^{s_0}$. Therefore, we come to the expression

$$\langle \tau_r \rangle = 1/M(\gamma\theta)^{s_0} \tag{21}$$

Although the estimation (21) is crude, it predicts a correct value of $\langle \tau_r \rangle$ for the model parameters used in Fig. 12, as well as it demonstrates a right tendency for variation of the hysteretic loop with θ.

Eq. (21) shows that the infinite system should have no hysteresis at all. This simply is the consequence of one-dimensionality of the model. Indeed, at $\alpha < \theta$ for any small but nonzero probability of creation of the s_0-atomic jam, at least one jam will certainly be created per each time step, and this jam will cause the RS $\rightarrow$ JS transition.

On the other hand, the behavior of a finite system is more interesting. According to Eq. (21), the timelife of the RS is $\tau_r \propto M^{-1}$, while the timelife of the JS is $\tau \propto M^2$ as follows from Eq. (8). Therefore, at $\alpha < \theta$ the state of a finite system should be bistable, time to time the chain should jump from RS to JS and back. The simulation results shown in Fig. 13 confirms this prediction.
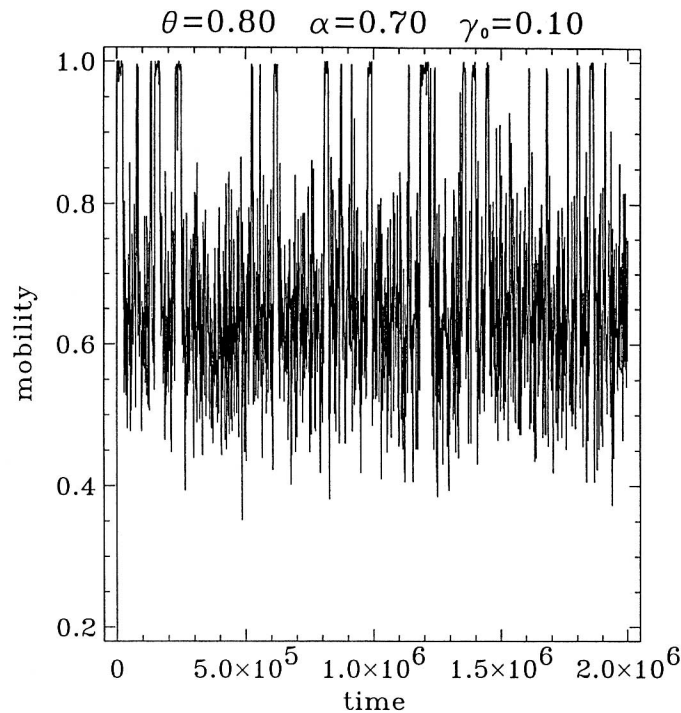

Fig. 13.   Dependence $B(t)$ for the model C. The system size is $M = 300$, $\theta = 0.8$, $\alpha = 0.7$, and $\gamma_0 = 0.1$.

5. CONCLUSIONS

Thus, we developed the new variant of the "traffic jam" model which successfully simulates the behavior of a more realistic but much more complicated FK model[1]. This model has a nonlinear dependence of mobility on the jump probability α, exhibits hysteresis, and describes the kinetics of organization of thermalized atoms into compact domains (jams). At the same time, being much simpler that the FK model, the LG traffic-jam model allows to simulate much larger systems on a much longer time scale, and also to develop the MF theory which explains the simulation results with a reasonable accuracy.

However, in difference with the FK model, the LG traffic-jam model studied in the present work, has no interatomic interaction except the trivial hard-core repulsion. As a consequence, the LG model cannot simulate a concerted motion of atoms of the FK model, the atomic jumps in the LG model are not correlated. Both these factor, the interatomic interaction and the concerted atomic motion, should increase the stability of the locked and running states, so that in the FK model the hysteresis exists in a much wider parameter range.

Finally, mention that the model studied in the present work, is the one-dimensional model. The percolation threshold in one-dimensional models is zero, i.e., even a single defect (jam in the present model) totally kills the conductivity (the running state). Which of the features of the model behavior will persist in a two-dimensional model, is the subject of a next work.

ACKNOWLEGMENTS

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